試卷一

解	分	備註
1. $\dfrac{(a^{-2})^3}{(a^4b^{-1})^2}$		
$=\dfrac{a^{-6}}{a^8b^{-2}}$	1M	給 $(xy)^m=x^my^m$ 或 $(x^m)^n=x^{mn}$
$=\dfrac{b^2}{a^{8+6}}$	1M	給 $z^{-p}=\dfrac{1}{z^p}$ 或 $\dfrac{z^p}{z^q}=z^{p-q}$
$=\dfrac{b^2}{a^{14}}$	1A	
	---------(3)	
2. $r=3+\dfrac{5p}{q-2}$		
$r(q-2)=3(q-2)+5p$	1M	
$rq-2r=3q-6+5p$		
$(r-3)q=5p+2r-6$	1M	給將 q 放在一邊
$q=\dfrac{5p+2r-6}{r-3}$	1A	或等價
$r-3=\dfrac{5p}{q-2}$	1M	
$q-2=\dfrac{5p}{r-3}$	1M	
$q=\dfrac{5p}{r-3}+2$	1A	或等價
$=\dfrac{5p+2r-6}{r-3}$		
	---------(3)	
3. (a) $9x^2-12xy+4y^2$		
$=(3x-2y)^2$	1A	
(b) $9x^2-12xy+4y^2-21x+14y$		
$=(3x-2y)^2-7(3x-2y)$	1M	給利用 (a) 的結果
$=(3x-2y)(3x-2y-7)$	1A	或等價
	---------(3)	
4. 設老師有 x 人，則學生有 $8x$ 人。	1M	或學生 x 人，老師 $\dfrac{1}{8}x$ 人
按題意，得		
$72x+60\times 8x=2208$	1M	
$552x=2208$		
$x=4$	1A	
老師及學生的總人數 $=4\times 9=36$	1A	
	---------(4)	

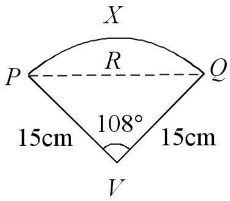
解	分	備註
5. (a) $\frac{2(x+3)}{5} > -2x-9$		
$2x+6 > -10x-45$		
$12x > -51$	1M	給將 x 放在一邊
$x > \frac{-17}{4}$		$x > -4.25$
或 $2-x \le 5$		
$-x \le 3$		
$x \ge -3$	1A	
因此 (*) 的解為 $x > \frac{-17}{4}$ 。	1A	$x > -4.25$
(b) -4	1A	
	---------(4)	
6. (a) A' 的坐標為 $(-3, -4)$ 。	1A	接受 $A'(-3, -4)$ 或 $A' = (-3, -4)$
B' 的坐標為 $(6, 8)$ 。	1A	
(b) $A'O$ 的斜率 $= \frac{0+4}{0+3} = \frac{4}{3}$	1M	接受 $m_{A'O} = \frac{4}{3}$；任何一項
$B'O$ 的斜率 $= \frac{8-0}{6-0} = \frac{4}{3}$		
$\because$ $A'O$ 的斜率 $= B'O$ 的斜率，		
又 O 為公共點，		
$\therefore$ $A'OB'$ 成一直線。	1	必須顯示理由
$A'O = \sqrt{(0+3)^2 + (0+4)^2} = 5$		任何一項
$B'O = \sqrt{(6-0)^2 + (8-0)^2} = 10$	1M	
$A'B' = \sqrt{(6+3)^2 + (8+4)^2} = 15$		
$\because$ $A'B' = A'O + B'O$		
$\therefore$ $A'OB'$ 成一直線	1	必須顯示理由
	---------(4)	
7. (a) $\frac{x+25}{100} = \frac{2}{5}$	1M	
$x = 15$	1A	
(b) 獲 B 操行學生的百分率		
$= 100\% - 25\% - 32\% - 15\%$		
$= 28\%$		
該校學生的人數 $= 350 \times \frac{100}{28}$	1M	
$= 1250$	1A	
	---------(4)	

解	分	備註
8. (a) $f(x)=a+bx^2$ ，其中 a 及 b 均為非零的常數。	1A	
$f(2)=a+4b=23$	1M	給任何一項代入
$f(7)=a+49b=563$		
解 (1) 及 (2) 兩式，得 $a=-25$ ， $b=12$	1A	給兩項正確
$\therefore\ f(x)=-25+12x^2$		
(b) $-25+12x^2=20x$		
$12x^2-20x-25=0$	1M	
$(2x-5)(6x+5)=0$		
$\therefore\ x=\frac{5}{2}$ 或 $x=-\frac{5}{6}$	1A	給兩項正確
	---------(5)	
9. (a) 最大絕對誤差 $=5$ mg		
最小可取重量		
$=500-5$	1M	
$=495$ (mg)	1A	
(b) 160 粒標準藥丸的最小可取重量		
$=(495)(160)$ mg	1M	
$=79200$ mg		給任何一項
$=79.2$ g		
<79.35 g	1A	
又 160 粒標準藥丸的最大可取重量		
$=(505)(160)$ mg		給任何一項
$=80800$ mg		
$=80.8$ g		
>79.45 g		
$\therefore$ 同意該宣稱。	1A	必須顯示理由
120 粒每粒重 495 mg 的標準藥丸和 40 粒每粒重 500 mg 的標準藥丸的總重量	1M	
$=495\times120+500\times40$ (mg)	1A	
$=79400$ mg		
$=79.4$ g		
$\therefore$同意該宣稱。	1A	必須顯示理由
	---------(5)	

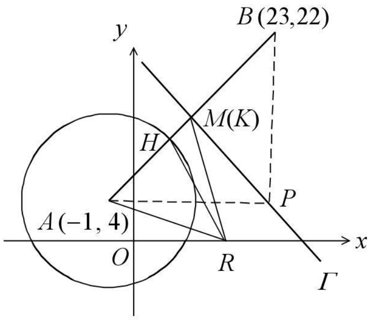
解	分	備註
10. (a) $\frac{1}{16}[40+a+42+45\times 2+46+48\times 2+50+53+54+55$ $+56+58+62+68+60+b]=52.5$	1M	$\frac{830+a+b}{16}=52.5$
即 $a+b=10$ ---(1)		
又 $60+b-(40+a)=28$	1M	
即 $b-a=8$ ---(2)		
解 (1)、(2) 兩式，得 $a=1$ ， $b=9$	1A	給兩項全對
標準差 ≈ 8.284020763 kg		
≈ 8.28 kg	1A	接受答案準確至 8.28 kg
	---------(4)	
(b) 所求概率 $=\frac{1+5+7}{16\times 15}\times 2$	1M	給分子或分母或數字 2 正確
$=\frac{13}{120}$	1A	接受答案準確至 0.108
利用列表法 (圖略)		
所求概率 $=\frac{26}{16\times 15}$	1M	給分子或分母正確
$=\frac{13}{120}$	1A	接受答案準確至 0.108
	---------(2)	

解			分	備註
11.	(a)	在 ΔABM 及 ΔADN 中		
		$\because$ $AB = AD$ [菱形性質]		或菱形的邊
		$\angle AMB = \angle AND = 90°$ [已知]		
		$\angle ABM = \angle ADN$ [菱形性質]		或菱形的對角
		$\therefore$ $\Delta ABM \cong \Delta ADN$ [AAS]		
		評分標準：		
		情況 1 附有正確理由的任何正確證明。	2	
		情況 2 未附有正確理由的任何正確證明。	1	
			---------(2)	
	(b)	$\because$ $\Delta ABM \cong \Delta ADN$		
		$\therefore$ $\angle BAM = \angle DAN$	1	
		又 $\angle ABD = \angle ADB$		
		及 $AB = AD$		
		$\therefore$ $\Delta ABE \cong \Delta ADF$		
		$\therefore$ $AE = AF$	1	必須顯示理由
			---------(2)	
	(c)	$\angle ABM = 180° - 90° - 20° = 70°$		
		$\angle ABE = \frac{1}{2} \times 70° = 35°$		
		$\angle AEF = 20° + 35° = 55°$	1A	
		作 $AR \perp EF$ 使垂足為 R，		
		則 $EF = 2ER$		
		$= 2 \times 12 \cos 55° \text{ (cm)}$	1M	
		$\approx 13.76583447 \text{ (cm)}$		
		$\approx 13.8 \text{ (cm)}$	1A	接受答案準確至 13.8 cm
		$\angle EAF = 180° - 2 \times 55° = 70°$		
		$EF = \sqrt{12^2 + 12^2 - 2(12)(12)\cos 70°}$	1M	或等價
		$\approx 13.8 \text{ (cm)}$	1A	接受答案準確至 13.8 cm
		$\angle EAF = 180° - 2 \times 55° = 70°$		
		$EF = \frac{12 \sin 70°}{\sin 55°}$	1M	或等價
		$\approx 13.8 \text{ (cm)}$	1A	接受答案準確至 13.8 cm
			---------(3)	

解	分	備註
12. (a) 設圓錐的底半徑為 r cm，		
則 $2\pi \times 15 \times \frac{216}{360} = 2\pi r$	1M	
$r = 9$		
在 $rt.\Delta VOX$ 中，		
$VO^2 = 15^2 - 9^2$		
$VO = 12$ (cm)	1A	
水的高度 $= 12 \times \frac{2}{3} = 8$ (cm)		
水面的半徑 $= 9 \times \frac{8}{12} = 6$ (cm)		
卡紙為水浸濕的面積 $= \pi \times 6 \times \sqrt{8^2 + 6^2}$	1M	給代入公式 $\pi r l$
$= 60\pi$ (cm^2)	1A	
	---------(4)	
(b) 半圓錐 $VPXQ$ 展開後如右圖。		
最短可能的路徑為線段 PRQ。	1M	
由圖知，弦 $PRQ <$ 弧 PXQ。	1A	
當螞蟻沿最短可能的路徑 PRQ 爬行時，其爬行路徑的長度有可能少於半圓 PXQ 的長度。	1A	必須顯示理由
弦 PRQ 的長度 $= 2 \times 15 \sin \frac{108°}{2}$		
≈ 24.27050983 (cm)	1A	接受答案準確至 24.3 cm
半圓 PXQ 的長度 $= \pi \times 9$		
≈ 28.27433388 (cm)		
$>$ 弦 PRQ 的長度		
當螞蟻沿最短可能的路徑 PRQ 爬行時，其爬行路徑的長度有可能少於半圓 PXQ 的長度。	1A	必須顯示理由
	---------(3)	



解	分	備註
13. (a) $f(x)=6x^3+5x^2+kx-8$		
$\equiv(3x^2+ax+2)(2x+b)+cx-6$	1M	
比較常數項，得 $2b-6=-8$	1M	給任何一項
$b=-1$		
比較 x^2 項，$3b+2a=5$		給兩項正確
$-3+2a=5$		
$a=4$	1A	
$f(x)=(3x^2+ax+2)(2x+b)+cx-6$	1M	
$=6x^3+3bx^2+2ax^2+abx+4x+2b+cx-6$		
$=6x^3+(3b+2a)x^2+(ab+c+4)x+2b-6$		
留意 $f(x)=6x^3+5x^2+kx-8$		
比較常數項及 x^2 項，得		
$2b-6=-8$	1M	給任何一項
$b=-1$		
$3b+2a=5$		給兩項正確
$a=4$	1A	
	---------(3)	
(b) (i) $g(x)=(3x^2+ax+2)(px+q)+cx-6$		
其中 p 、 q 為常數，且 $p\neq0$ 。		
$f(x)-g(x)=(3x^2+ax+2)(2x+b)-(3x^2+ax+2)(px+q)$	1M	
$=(3x^2+ax+2)(2x+b-px-q)$		
$\therefore$ $f(x)-g(x)$ 可被 $3x^2+ax+2$ 整除。	1	必須顯示理由
(ii) 當 $f(x)-g(x)=0$		
$(3x^2+4x+2)(2x-px-1-q)=0$		
(據 (a) 及 (b)(i))		
對 $3x^2+4x+2=0$		
$\because\Delta=4^2-4(3)(2)=-8<0$	1M	
$\therefore$ $3x^2+4x+2=0$ 的根不是實數。		
因此，不是所有 $f(x)-g(x)=0$ 的根都是實數。		
故不同意該宣稱。	1A	必須顯示理由
	---------(4)	

解	分	備註
14. (a) $C: x^2+y^2+2x-8y-83=0$		
圓心 A 的坐標為 $(-1, 4)$	1A	接受 $A(-1, 4)$ 或 $A=(-1, 4)$
圓 C 的半徑為 $\frac{1}{2}\sqrt{2^2+(-8)^2-4(-83)}=10$	1A	接受 $r=10$
	---------(2)	
(b) 		
(i) 設 P 點坐標為 (x, y) 。		接受設 $P(x, y)$
由 $PA=PB$，得		
$\sqrt{(x+1)^2+(y-4)^2}=\sqrt{(x-23)^2+(y-22)^2}$	1M	
$(x+1)^2+(y-4)^2=(x-23)^2+(y-22)^2$		
$x^2+2x+1+y^2-8y+16=x^2-46x+529+y^2-44y+484$	1M	給左或右展式正確
$48x+36y-996=0$		
即 Γ 的方程為 $4x+3y-83=0$	1A	
Γ 為線段 AB 的垂直平分線。		
AB 的斜率 $=\frac{22-4}{23+1}=\frac{3}{4}$		接受 $m_{AB}=\frac{3}{4}$
Γ 為斜率 $=-\frac{4}{3}$	1M	
AB 的中點 $M=(11, 13)$		
Γ 的方程為 $\frac{y-13}{x-11}=-\frac{4}{3}$	1M	
即 $4x+3y-83=0$	1A	或等價
(ii) 留意 K 點即 M 點 (AB 的中點)，又 AHK 成一直線。	1M	$K=(11, 13)$
$AK=\sqrt{(11+1)^2+(13-4)^2}$		
$=15$		
$AH=10$		
$\therefore HK=5$		
ΔRAH 與 ΔRHK 面積之比 $=AH:HK$	1M	
$=10:5$		
$=2:1$	1A	
	---------(6)	

解	分	備註
15. $y=\frac{a}{b^x}$		
代入點 (1, 3) 及點 $(4,\frac{1}{9})$，得		
$\frac{a}{b}=3$ ……(1)		
$\frac{a}{b^4}=\frac{1}{9}$ ……(2)		
(1)÷(2)，得 $b^3=27$	1M	
$b=3$		
代入 (1)，得 $a=9$	1A	給兩項正確
$\therefore\ y=\frac{9}{3^x}$		
$y=3^{2-x}$		
$\log_3 y=2-x$ 　　 $\log y=(2-x)\log 3$	1M	
$x=2-\log_3 y$ 　　 $x=\frac{2\log 3-\log y}{\log 3}$	1A	或 $x=\frac{\log 9-\log y}{\log 3}$ 或 $x=2-\frac{\log y}{\log 3}$ 或等價
	---------(4)	
16. (a) 所求概率 $=\frac{8}{8}\times\frac{1}{8}\times\frac{7}{8}\times\frac{1}{8}\times\frac{C_2^4}{2!}$	1M	給 $\frac{8}{8}\times\frac{1}{8}\times\frac{7}{8}\times\frac{1}{8}$
$=\frac{21}{512}$	1A	接受答案準確至 0.0410
所求概率 $=(\frac{1}{8})^4C_2^8C_2^4$	1M	給 $(\frac{1}{8})^4C_2^8$
$=\frac{21}{512}$	1A	接受答案準確至 0.0410
	---------(2)	
(b) 所求概率 $=\frac{21}{512}+\frac{8}{8}\times\frac{1}{8}\times\frac{1}{8}\times\frac{7}{8}\times C_1^4$	1M+1A	1M 給 $P_1+\frac{8}{8}\times\frac{1}{8}\times\frac{1}{8}\times\frac{7}{8}$
$=\frac{49}{512}$	1A	接受答案準確至 0.0957
所求概率		
$=1-\frac{8}{8}\times\frac{1}{8}\times\frac{1}{8}\times\frac{1}{8}-\frac{8}{8}\times\frac{1}{8}\times\frac{7}{8}\times\frac{6}{8}\times C_2^4-\frac{8}{8}\times\frac{7}{8}\times\frac{6}{8}\times\frac{5}{8}$	1M+1A	1M 給 $1-P_1-P_2-P_3$
$=\frac{49}{512}$	1A	接受答案準確至 0.0957
所求概率 $=(C_2^8)(2^4-2)\times\frac{1}{8^4}$	1M+1A	1M 給 $C_2^8\times\frac{1}{8^4}$
$=\frac{49}{512}$	1A	接受答案準確至 0.0957
	---------(3)	

	解	分	備註
17.	(a) $T(n)=3^{4n-1}$		
	$T(1), T(2), T(3), \cdots, T(n)$ 成一等比數列，		或成一幾何數列或 G.S.
	首項 $T(1)=3^3=27$		接受 $a=27$ ， $r=81$
	公比 $=3^4=81$		
	$T(1)+T(2)+T(3)+\cdots+T(n)$		
	$=\dfrac{27(81^n-1)}{81-1}$	1M	或 $\dfrac{27(1-81^n)}{1-81}$
	$=\dfrac{27(81^n-1)}{80}$	1A	
		---------(2)	
	(b) $T(1)T(2)T(3)\cdots T(n)$		
	$=3^3\cdot3^7\cdot3^{11}\cdots\cdots3^{4n-1}$		
	$=3^{3+7+11+\cdots+(4n-1)}$	1M	
	$=3^{(3+4n-1)\cdot\frac{n}{2}}$		
	$=3^{(1+2n)n}$		
	由於 $T(1)T(2)T(3)\cdots T(n)\geq3^{630}$		
	故 $3^{(1+2n)n}\geq3^{630}$		
	$(1+2n)n\geq630$	1M	
	$2n^2+n-630\geq0$		
	$(2n-35)(n+18)\geq0$		
	$\therefore\ n\leq-18$ (捨) 或 $n\geq\dfrac{35}{2}$		
	$\therefore n$ 的最小值為 18。	1A	
		---------(3)	

解	分	備註
18. (a) $y=2x^2-4kx+k^2-1$		
$=2(x^2-2kx+k^2-k^2)+k^2-1$	1M	必須顯示配方過程
$=2(x-k)^2-k^2-1$		
P 點的坐標為 $(k,-k^2-1)$	1A	不接受直接寫答案
	--------(2)	
(b) $2x^2-4kx+k^2-1=15$		
$2x^2-4kx+k^2-16=0$		
$\because\ \Delta=(-4k)^2-8(k^2-16)$	1M	
$=8k^2+128$		
>0		
$\therefore$ L 與 Γ 相交於兩相異點。	1	必須顯示理由
$\because$ 拋物線 Γ 的頂點 P 的 y 坐標 $=-k^2-1$		
<0	1M	至少有兩項正確
而 $L: y=15$ 為一位於 x 軸上方的水平線，		
又拋物線的開口向上。		
$\therefore$ L 與 Γ 相交於兩相異點。	1	必須顯示理由
	--------(2)	
(c) (i) a、b 為方程 $2x^2-4kx+k^2-16=0$ 的兩根。		
$\therefore\ a+b=-\dfrac{-4k}{2}=2k$	1M	給兩項正確
$ab=\dfrac{k^2-16}{2}$		
$(a-b)^2=a^2+b^2-2ab$		
$=(a+b)^2-4ab$		
$=(2k)^2-4(\dfrac{k^2-16}{2})$		
$=2k^2+32$	1A	
(ii) ΔPAB 的高 $=15-(-k^2-1)$		
$=k^2+16$		
≥ 16	1M	給任何一項
又 $AB^2=2k^2+32$		
>25 $\quad(\geq 32)$		
即 底 $AB>5$ $\quad(\geq\sqrt{32})$		
$\therefore$ ΔPAB 的面積 $>\dfrac{16\times 5}{2}$ $\quad(\geq\dfrac{16\sqrt{32}}{2})$		
$=40$ $\quad(=32\sqrt{2})$		
因此，ΔPAB 的面積沒有可能小於 40。	1A	必須顯示理由
	--------(4)	

解	分	備註
19. (a) $AA' = 2\sin 30° = 1$		
$AB = \dfrac{1}{\sin 45°}$		
≈ 1.414213562		
≈ 1.41 (m) $\quad (\sqrt{2}$ m)	1A	接受答案準確至 1.41 m
在 ΔABC 中，藉餘弦公式得		
$BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos\angle BAC$	1M	
$\approx 2^2 + 1.414213562^2 - 2(2)(1.414213562)\cos 60°$		
$BC \approx 1.780891034$		
≈ 1.78 (m) $\quad (\sqrt{6-2\sqrt{2}})$	1A	接受答案準確至 1.78 m
	---------(3)	
(b) (i) $BB' \approx 1.414213562 \sin 30°$		
$\approx 0.707106781 \quad (\dfrac{\sqrt{2}}{2})$	1A	給任何一項
$CC' = 2\sin 45°$		
$\approx 1.414213562 \quad (\sqrt{2})$		
作 $BN \perp CC'$ 使垂足為 N。		
$CN \approx 1.414213562 - 0.707106781$		
$\approx 0.707106781 \quad (\dfrac{\sqrt{2}}{2})$		
$B'C' = BN$		
$= \sqrt{BC^2 - CN^2}$	1M	
$\approx \sqrt{1.780891034^2 - 0.707106781^2}$		
≈ 1.634494685		
≈ 1.63 (m) $\quad (\sqrt{\dfrac{11-4\sqrt{2}}{2}}$ m)	1A	接受答案準確至 1.63 m
(ii) $AB' = AB\cos 30°$		
$\approx 1.414213562 \cos 30°$		
$\approx 1.224744871 \quad (\dfrac{\sqrt{6}}{2})$		
$AC' = AC\cos 45°$		
$= 2\cos 45°$		
$\approx 1.414213562 \quad (\sqrt{2})$		
在 $\Delta AB'C'$ 中，藉餘弦公式得		
$\cos\angle B'AC' = \dfrac{(AB')^2 + (AC')^2 - (B'C')^2}{2(AB')(AC')}$	1M	
$\approx \dfrac{1.224744871^2 + 1.414213562^2 - 1.634494685^2}{2(1.224744871)(1.414213562)}$		$0.239146311 \quad (\dfrac{\sqrt{2}-1}{\sqrt{3}})$
$\angle B'AC' \approx 76.16383955°$		
$\approx 76.2°$	1A	接受答案準確至 76.2°
影子 $B'AC'$ 的面積		
$\approx \dfrac{1}{2}(1.224744871)(1.414213562)\sin 76.16383955°$	1M	
≈ 0.840896414		
≈ 0.841 (m^2)	1A	接受答案準確至 0.841 m^2

解	分	備註
$\Delta B'AC'$ 的半周界 $\approx \dfrac{1.224744871+1.634494685+1.414213562}{2}$ ≈ 2.136726559 (記為 s) 影子 $B'AC'$ 的面積 $\approx \sqrt{s(s-AB')(s-AC')(s-B'C')}$ ≈ 0.840896414 ≈ 0.841 (m^2)	 1M 1A	 接受答案準確至 0.841 m^2
(iii) ΔABC 的面積 $=\dfrac{1}{2}(AB)(AC)\sin 60°$ $\approx \dfrac{1}{2}(1.414213562)(2)\sin 60°$ $\left(\dfrac{1}{2}(\sqrt{2})(2)\sin 60°\right)$ ≈ 1.224744871 $\left(\dfrac{\sqrt{6}}{2}\right)$ 設所求傾角為 θ， 則 $\cos\theta = \dfrac{\Delta B'AC' \text{的面積}}{\Delta ABC \text{的面積}}$ $\approx \dfrac{0.840896414}{1.224744871}$ $\theta \approx 46.6392948°$ $> 45°$ 因此，不同意該宣稱。	 1M 1M 1A -------(10)	 必須顯示理由

Paper 1

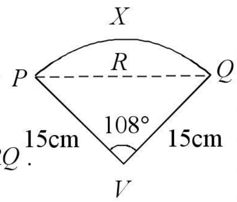
Solution	Marks	Remarks
1. $\dfrac{(a^{-2})^3}{(a^4b^{-1})^2}$		
$= \dfrac{a^{-6}}{a^8b^{-2}}$	1M	for $(xy)^m = x^m y^m$ or $(x^m)^n = x^{mn}$
$= \dfrac{b^2}{a^{8+6}}$	1M	for $z^{-p} = \dfrac{1}{z^p}$ or $\dfrac{z^p}{z^q} = z^{p-q}$
$= \dfrac{b^2}{a^{14}}$	1A	
	---------(3)	
2. $r = 3 + \dfrac{5p}{q-2}$		
$r(q-2) = 3(q-2) + 5p$	1M	
$rq - 2r = 3q - 6 + 5p$		
$(r-3)q = 5p + 2r - 6$	1M	for putting q on one side
$q = \dfrac{5p+2r-6}{r-3}$	1A	or equivalent
$r - 3 = \dfrac{5p}{q-2}$	1M	
$q - 2 = \dfrac{5p}{r-3}$	1M	
$q = \dfrac{5p}{r-3} + 2$	1A	or equivalent
$= \dfrac{5p+2r-6}{r-3}$		
	---------(3)	
3. (a) $9x^2 - 12xy + 4y^2$		
$= (3x-2y)^2$	1A	
(b) $9x^2 - 12xy + 4y^2 - 21x + 14y$		
$= (3x-2y)^2 - 7(3x-2y)$	1M	for using the result of (a)
$= (3x-2y)(3x-2y-7)$	1A	or equivalent
	---------(3)	
4. Let x be the number of teachers ,		or let x be the number of students ,
then the number of students is $8x$.	1M	then the number of teachers is $\dfrac{1}{8}x$.
$72x + 60 \times 8x = 2208$	1M	
$552x = 2208$		
$x = 4$	1A	
The total number of teachers and students $= 4 \times 9 = 36$	1A	
	---------(4)	

	Solution	Marks	Remarks
5. (a)	$\frac{2(x+3)}{5} > -2x-9$		
	$2x+6 > -10x-45$		
	$12x > -51$	1M	for putting x on one side
	$x > \frac{-17}{4}$		$x > -4.25$
	or $2-x \le 5$		
	$-x \le 3$		
	$x \ge -3$	1A	
	Thus, the solution of (*) is $x > \frac{-17}{4}$.	1A	$x > -4.25$
(b)	-4	1A	
		---------(4)	
6. (a)	The coordinates of A' are $(-3, -4)$.	1A	accept $A'(-3, -4)$ or $A' = (-3, -4)$
	The coordinates of B' are $(6, 8)$.	1A	
(b)	The slope of $A'O = \frac{0+4}{0+3} = \frac{4}{3}$	1M	accept $m_{A'O} = \frac{4}{3}$ (either one)
	The slope of $B'O = \frac{8-0}{6-0} = \frac{4}{3}$		(either one)
	$\because$ The slope of $A'O$ = the slope of $B'O$		
	Also, O is a common point,		
	$\therefore$ $A'OB'$ is a straight line .	1	f.t.
	$A'O = \sqrt{(0+3)^2+(0+4)^2} = 5$		(either one)
	$B'O = \sqrt{(6-0)^2+(8-0)^2} = 10$	1M	either one
	$A'B' = \sqrt{(6+3)^2+(8+4)^2} = 15$		(either one)
	$\because$ $A'B' = A'O + B'O$		
	$\therefore$ $A'OB'$ is a straight line .	1	f.t.
		---------(4)	
7. (a)	$\frac{x+25}{100} = \frac{2}{5}$	1M	
	$x = 15$	1A	
(b)	Percentage of students got conduct in grade B		
	$= 100\% - 25\% - 32\% - 15\%$		
	$= 28\%$		
	Number of students in the school $= 350 \times \frac{100}{28}$	1M	
	$= 1250$	1A	
		---------(4)	

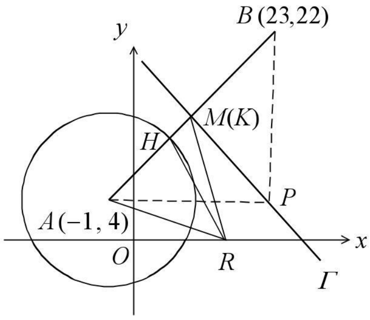
Solution	Marks	Remarks
8. (a) Let $f(x) = a + bx^2$, where a and b are non-zero constants .	1A	
$f(2) = a + 4b = 23$	1M	for either substitution
$f(7) = a + 49b = 563$		
Solving (1) and (2) , we have $a = -25$ and $b = 12$	1A	for both correct
$\therefore\ f(x) = -25 + 12x^2$		
(b) $-25 + 12x^2 = 20x$		
$12x^2 - 20x - 25 = 0$	1M	
$(2x - 5)(6x + 5) = 0$		
$\therefore\ x = \dfrac{5}{2}$ or $x = -\dfrac{5}{6}$	1A	for both correct
	---------(5)	
9. (a) The maximum absolute error $= 5$ mg		
The least possible weight		
$= 500 - 5$	1M	
$= 495$ (mg)	1A	
(b) The least possible total weight of 160 *standard* pills		
$= (495)(160)$ mg	1M	either one
$= 79200$ mg		
$= 79.2$ g		
< 79.35 g	1A	either one
Also, the greatest possible total weight of 160 *standard* pills		
$= (505)(160)$ mg		(either one, with $(495)(160)$ mg)
$= 80800$ mg		
$= 80.8$ g		
> 79.45 g		(either one, with < 79.35 g)
Thus , the claim is agreed .	1A	f.t.
The total weight of 120 *standard* pills each measured as 495 mg and 40 *standard* pills each measured as 500 mg	1M	
$= 495 \times 120 + 500 \times 40$ (mg)	1A	
$= 79400$ mg		
$= 79.4$ g		
Thus , the claim is agreed .	1A	f.t.
	---------(5)	

Solution	Marks	Remarks
10. (a) $\frac{1}{16}[40+a+42+45\times 2+46+48\times 2+50+53+54+55$ $+56+58+62+68+60+b]=52.5$	1M	$\frac{830+a+b}{16}=52.5$
i.e. $a+b=10 \quad ---(1)$		
Also , $60+b-(40+a)=28$	1M	
i.e. $b-a=8 \quad ---(2)$		
Solving (1) and (2) , we have $a=1$ and $b=9$	1A	for both correct
The standard deviation ≈ 8.284020763 kg		
≈ 8.28 kg	1A	r.t. 8.28 kg
	---------(4)	
(b) The required probability $=\frac{1+5+7}{16\times 15}\times 2$	1M	for numerator or denominator or the multiplier 2
$=\frac{13}{120}$	1A	r.t. 0.108
Consider the tabular method , the required probability $=\frac{26}{16\times 15}$	1M	for numerator or denominator
$=\frac{13}{120}$	1A	r.t. 0.108
	---------(2)	

		Solution	Marks	Remarks
11.	(a)	In ΔABM and ΔADN ,		
		$\because\ AB = AD$ [property of rhombus]		or sides of rhombus
		$\angle AMB = \angle AND = 90°$ [given]		
		$\angle ABM = \angle ADN$ [property of rhombus]		or opp. angles of rhombus
		$\therefore\ \Delta ABM \cong \Delta ADN$ [AAS]		
		Marking Scheme :		
		Case 1 Any correct proof with correct reasons .	2	
		Case 2 Any correct proof without reasons .	1	
			---------(2)	
	(b)	$\because\ \Delta ABM \cong \Delta ADN$		
		$\therefore\ \angle BAM = \angle DAN$	1	
		Also , $\angle ABD = \angle ADB$		
		and $AB = AD$		
		$\therefore\ \Delta ABE \cong \Delta ADF$		
		$\therefore\ AE = AF$	1	f.t.
			---------(2)	
	(c)	$\angle ABM = 180° - 90° - 20° = 70°$		
		$\angle ABE = \frac{1}{2} \times 70° = 35°$		
		$\angle AEF = 20° + 35° = 55°$	1A	
		Construct $AR \perp EF$ such that R is the foot of the perpendicular .		
		Then $EF = 2ER$		
		$= 2 \times 12 \cos 55°$ (cm)	1M	
		≈ 13.76583447 (cm)		
		≈ 13.8 (cm)	1A	r.t. 13.8 cm
		$\angle EAF = 180° - 2 \times 55° = 70°$		
		$EF = \sqrt{12^2 + 12^2 - 2(12)(12)\cos 70°}$	1M	or equivalent
		≈ 13.8 (cm)	1A	r.t. 13.8 cm
		$\angle EAF = 180° - 2 \times 55° = 70°$		
		$EF = \frac{12 \sin 70°}{\sin 55°}$	1M	or equivalent
		≈ 13.8 (cm)	1A	r.t. 13.8 cm
			---------(3)	

		Solution	Marks	Remarks
12.	(a)	Let r cm be the base radius of the vessel .		
		$2\pi \times 15 \times \frac{216}{360} = 2\pi r$	1M	
		$r = 9$		
		In $rt.\Delta VOX$,		
		$VO^2 = 15^2 - 9^2$		
		$VO = 12$ (cm)	1A	
		The depth of the water $= 12 \times \frac{2}{3} = 8$ (cm)		
		The radius of the water surface $= 9 \times \frac{8}{12} = 6$ (cm)		
		The area of the wet part of the vessel $= \pi \times 6 \times \sqrt{8^2 + 6^2}$	1M	for $\pi r l$
		$= 60\pi$ (cm ^{2})	1A	
			---------(4)	
	(b)	Cut the vessel vertically along PQ , then unfold the paper $VPXQ$ as shown . 		
		The shortest path is the line segment PRQ .	1M	
		Refer to the figure,		
		chord length PRQ < arc length PXQ	1A	
		When the ant walks along the shortest path PRQ , it is possible that the distance walked by the ant is less than the length of the semi-circle PXQ .	1A	f.t.
		The length of the chord $PRQ = 2 \times 15 \sin \frac{108°}{2}$		
		≈ 24.27050983 (cm)	1A	r.t. 24.3 cm
		The length of the semi-circle $PXQ = \pi \times 9$		
		≈ 28.27433388 (cm)		
		> chord length PRQ		
		When the ant walks along the shortest path PRQ , it is possible that the distance walked by the ant is less than the length of the semi-circle PXQ .	1A	f.t.
			---------(3)	

Solution	Marks	Remarks
13. (a) $\mathrm{f}(x)=6x^3+5x^2+kx-8$		
$\equiv(3x^2+ax+2)(2x+b)+cx-6$	1M	
By comparing the constant term , $2b-6=-8$	1M	either one
$b=-1$		
By comparing the x^2 term , $3b+2a=5$		for both correct
$-3+2a=5$		
$a=4$	1A	
$\mathrm{f}(x)=(3x^2+ax+2)(2x+b)+cx-6$	1M	
$=6x^3+3bx^2+2ax^2+abx+4x+2b+cx-6$		
$=6x^3+(3b+2a)x^2+(ab+c+4)x+2b-6$		
Note that $\mathrm{f}(x)=6x^3+5x^2+kx-8$		
By comparing the constant term and the x^2 term ,		
$2b-6=-8$	1M	either one
$b=-1$		
$3b+2a=5$		for both correct
$a=4$	1A	
	---------(3)	
(b) (i) $g(x)=(3x^2+ax+2)(px+q)+cx-6$, where p and q are constants , and $p\neq 0$.		
$\mathrm{f}(x)-\mathrm{g}(x)=(3x^2+ax+2)(2x+b)-(3x^2+ax+2)(px+q)$	1M	
$=(3x^2+ax+2)(2x+b-px-q)$		
$\therefore$ $\mathrm{f}(x)-\mathrm{g}(x)$ is divisible by $3x^2+ax+2$.	1	f.t.
(ii) When $\mathrm{f}(x)-\mathrm{g}(x)=0$,		
$(3x^2+4x+2)(2x-px-1-q)=0$ (from (a) and (b)(i))		
For $3x^2+4x+2=0$		
$\because$ $\Delta=4^2-4(3)(2)=-8<0$	1M	
$\therefore$ $3x^2+4x+2=0$ has non-real roots .		
Therefore , not all the roots of $\mathrm{f}(x)-\mathrm{g}(x)=0$ are real numbers .		
Thus , the claim is disagreed .	1A	f.t.
	---------(4)	

		Solution	Marks	Remarks
14.	(a)	$C: x^2+y^2+2x-8y-83=0$		
		The coordinates of A are $(-1,4)$	1A	accept $A(-1,4)$ or $A=(-1,4)$
		The radius of C is $\frac{1}{2}\sqrt{2^2+(-8)^2-4(-83)}=10$	1A	accept $r=10$
			---------(2)	
	(b)	 		
		(i) Let (x,y) be the coordinates of point P.		accept $P(x,y)$
		Form $PA=PB$,		
		$\sqrt{(x+1)^2+(y-4)^2}=\sqrt{(x-23)^2+(y-22)^2}$	1M	
		$(x+1)^2+(y-4)^2=(x-23)^2+(y-22)^2$		
		$x^2+2x+1+y^2-8y+16=x^2-46x+529+y^2-44y+484$	1M	for correct expansion of one side
		$48x+36y-996=0$		
		The equation of Γ is $4x+3y-83=0$.	1A	or equivalent
		Γ is the perpendicular bisector of line segment AB .		
		The slope of $AB=\frac{22-4}{23+1}=\frac{3}{4}$		accept $m_{AB}=\frac{3}{4}$
		The slope of $\Gamma=-\frac{4}{3}$	1M	
		Mid-point of AB is $M=(11,13)$		
		The equation of Γ is $\frac{y-13}{x-11}=-\frac{4}{3}$	1M	
		i.e. $4x+3y-83=0$	1A	
		(ii) Note that point K is the mid-point of AB, i.e. point M, and AHK is a straight line .	1M	$K=(11,13)$
		$AK=\sqrt{(11+1)^2+(13-4)^2}$		
		$=15$		
		$AH=10$		
		$\therefore\ HK=5$		
		The ratio of the areas ΔRAH to ΔRHK		
		$=AH:HK$	1M	
		$=10:5$		
		$=2:1$	1A	
			---------(6)	

Solution	Marks	Remarks
15. $y = \frac{a}{b^x}$		
Sub. the points $(1, 3)$ and $(4, \frac{1}{9})$,		
$\frac{a}{b} = 3$ ……(1)		
$\frac{a}{b^4} = \frac{1}{9}$ ……(2)		
$(1) \div (2)$, we have $b^3 = 27$	1M	
$b = 3$		for both correct
Sub. into (1), $a = 9$	1A	
$\therefore\ y = \frac{9}{3^x}$		
$y = 3^{2-x}$		
$\log_3 y = 2 - x$ $\log y = (2-x)\log 3$	1M	
$x = 2 - \log_3 y$ $x = \frac{2\log 3 - \log y}{\log 3}$	1A	or $x = \frac{\log 9 - \log y}{\log 3}$ or $x = 2 - \frac{\log y}{\log 3}$ or equivalent
	---------(4)	
16. (a) The required probability $= \frac{8}{8} \times \frac{1}{8} \times \frac{7}{8} \times \frac{1}{8} \times \frac{C_2^4}{2!}$	1M	for $\frac{8}{8} \times \frac{1}{8} \times \frac{7}{8} \times \frac{1}{8}$
$= \frac{21}{512}$	1A	r.t. 0.0410
The required probability $= (\frac{1}{8})^4 C_2^8 C_2^4$	1M	for $(\frac{1}{8})^4 C_2^8$
$= \frac{21}{512}$	1A	r.t. 0.0410
	---------(2)	
(b) The required probability $= \frac{21}{512} + \frac{8}{8} \times \frac{1}{8} \times \frac{1}{8} \times \frac{7}{8} \times C_1^4$	1M+1A	1M for $P_1 + \frac{8}{8} \times \frac{1}{8} \times \frac{1}{8} \times \frac{7}{8}$
$= \frac{49}{512}$	1A	r.t. 0.0957
The required probability $= 1 - \frac{8}{8} \times \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} - \frac{8}{8} \times \frac{1}{8} \times \frac{7}{8} \times \frac{6}{8} \times C_2^4 - \frac{8}{8} \times \frac{7}{8} \times \frac{6}{8} \times \frac{5}{8}$	1M+1A	1M for $1 - P_1 - P_2 - P_3$
$= \frac{49}{512}$	1A	r.t. 0.0957
The required probability $= (C_2^8)(2^4 - 2) \times \frac{1}{8^4}$	1M+1A	1M for $C_2^8 \times \frac{1}{8^4}$
$= \frac{49}{512}$	1A	r.t. 0.0957
	---------(3)	

	Solution	Marks	Remarks
17.	(a) $T(n)=3^{4n-1}$		
	$T(1), T(2), T(3), \cdots, T(n)$ is a geometric sequence		or G.S.
	First term $T(1)=3^3=27$		accept $a=27$ and $r=81$
	Common ratio $=3^4=81$		
	$T(1)+T(2)+T(3)+\cdots+T(n)$		
	$=\dfrac{27(81^n-1)}{81-1}$	1M	or $\dfrac{27(1-81^n)}{1-81}$
	$=\dfrac{27(81^n-1)}{80}$	1A	
	(b) $T(1)T(2)T(3)\cdots T(n)$		
	$=3^3\cdot3^7\cdot3^{11}\cdot\cdots\cdot3^{4n-1}$		
	$=3^{3+7+11+\cdots+(4n-1)}$	1M	
	$=3^{(3+4n-1)\cdot\frac{n}{2}}$		
	$=3^{(1+2n)n}$		
	From $T(1)T(2)T(3)\cdots T(n)\geq 3^{630}$,		
	$3^{(1+2n)n}\geq 3^{630}$		
	$(1+2n)n\geq 630$	1M	
	$2n^2+n-630\geq 0$		
	$(2n-35)(n+18)\geq 0$		
	$\therefore\ n\leq -18$ (rej.) or $n\geq\dfrac{35}{2}$		
	$\therefore$ The minimum value of n is 18 .	1A	
		---------(3)	

Solution	Marks	Remarks
18. (a) $y=2x^2-4kx+k^2-1$		
$=2(x^2-2kx+k^2-k^2)+k^2-1$	1M	show the working
$=2(x-k)^2-k^2-1$		
The coordinates of point P are $(k,-k^2-1)$.	1A	not accept write down the answer
	--------(2)	
(b) $2x^2-4kx+k^2-1=15$		
$2x^2-4kx+k^2-16=0$		
$\because\ \Delta=(-4k)^2-8(k^2-16)$	1M	
$=8k^2+128$		
>0		
$\therefore\ L$ and Γ intersect at two distinct points .	1	f.t.
$\because$ The y-coordinates of the vertex P of the parabola Γ		
$=-k^2-1$		
<0	1M	at least two correct
And $L: y=15$ is a horizontal line above the x-axis ,		
Also the parabola opens upwards .		
$\therefore\ L$ and Γ intersect at two distinct points .	1	f.t.
	--------(2)	
(c) (i) a , b are the roots of the equation $2x^2-4kx+k^2-16=0$		
$\therefore\ a+b=-\dfrac{-4k}{2}=2k$	1M	for both correct
$ab=\dfrac{k^2-16}{2}$		
$(a-b)^2=a^2+b^2-2ab$		
$=(a+b)^2-4ab$		
$=(2k)^2-4(\dfrac{k^2-16}{2})$		
$=2k^2+32$	1A	
(ii) The height of $\Delta PAB=15-(-k^2-1)$		
$=k^2+16$		
≥ 16	1M	either one
And $AB^2=2k^2+32$		
>25 $\quad(\geq 32)$		
i.e. the base $AB>5$ $\quad(\geq\sqrt{32})$		
$\therefore$ The area of $\Delta PAB>\dfrac{16\times 5}{2}$ $\quad(\geq\dfrac{16\sqrt{32}}{2})$		
$=40$ $\quad(=32\sqrt{2})$		
Therefore , it is impossible that the area of ΔPAB is less than 40 .	1A	f.t.
	--------(4)	

	Solution	Marks	Remarks
19.	(a) $AA' = 2\sin 30° = 1$		
	$AB = \dfrac{1}{\sin 45°}$		
	≈ 1.414213562		
	≈ 1.41 (m) $\quad (\sqrt{2}$ m)	1A	r.t. 1.41 m
	In ΔABC ,		
	by cosine formula ,		
	$BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos\angle BAC$	1M	
	$\approx 2^2 + 1.414213562^2 - 2(2)(1.414213562)\cos 60°$		
	$BC \approx 1.780891034$		
	≈ 1.78 (m) $\quad (\sqrt{6-2\sqrt{2}})$	1A	r.t. 1.78 m
		---------(3)	
	(b) (i) $BB' \approx 1.414213562 \sin 30°$		
	$\approx 0.707106781 \quad (\dfrac{\sqrt{2}}{2})$	1A	either one
	$CC' = 2\sin 45°$		
	$\approx 1.414213562 \quad (\sqrt{2})$		
	Construct $BN \perp CC'$ such that N is the foot of the perpendicular .		
	$CN \approx 1.414213562 - 0.707106781$		
	$\approx 0.707106781 \quad (\dfrac{\sqrt{2}}{2})$		
	$B'C' = BN$		
	$= \sqrt{BC^2 - CN^2}$	1M	
	$\approx \sqrt{1.780891034^2 - 0.707106781^2}$		
	≈ 1.634494685		
	≈ 1.63 (m) $\quad (\sqrt{\dfrac{11-4\sqrt{2}}{2}}$ m)	1A	r.t. 1.63 m
	(ii) $AB' = AB\cos 30°$		
	$\approx 1.414213562 \cos 30°$		
	$\approx 1.224744871 \quad (\dfrac{\sqrt{6}}{2})$		
	$AC' = AC\cos 45°$		
	$= 2\cos 45°$		
	$\approx 1.414213562 \quad (\sqrt{2})$		
	In $\Delta AB'C'$,		
	by cosine formula ,		
	$\cos\angle B'AC' = \dfrac{(AB')^2 + (AC')^2 - (B'C')^2}{2(AB')(AC')}$	1M	
	$\approx \dfrac{1.224744871^2 + 1.414213562^2 - 1.634494685^2}{2(1.224744871)(1.414213562)}$		0.239146311 $\quad (\dfrac{\sqrt{2}-1}{\sqrt{3}})$
	$\angle B'AC' \approx 76.16383955°$		
	$\approx 76.2°$	1A	r.t. 76.2°
	The area of the shadow $B'AC'$		
	$\approx \dfrac{1}{2}(1.224744871)(1.414213562)\sin 76.16383955°$	1M	
	≈ 0.840896414		
	≈ 0.841 (m^2)	1A	r.t. 0.841 m^2

Solution	Marks	Remarks
The semi-perimeter of $\Delta B'AC'$		
$\approx \dfrac{1.224744871 + 1.634494685 + 1.414213562}{2}$		
≈ 2.136726559 (denoted by s)		
The area of the shadow $B'AC'$		
$\approx \sqrt{s(s-AB')(s-AC')(s-B'C')}$	1M	
≈ 0.840896414		
≈ 0.841 (m^2)	1A	r.t. $0.841\ m^2$
(iii) The area of ΔABC		
$= \dfrac{1}{2}(AB)(AC)\sin 60°$		
$\approx \dfrac{1}{2}(1.414213562)(2)\sin 60°$ $\quad (\dfrac{1}{2}(\sqrt{2})(2)\sin 60°)$	1M	
≈ 1.224744871 $\quad (\dfrac{\sqrt{6}}{2})$		
Let θ be the required angle of inclination ,		
then $\cos\theta = \dfrac{\text{area of } \Delta B'AC'}{\text{area of } \Delta ABC}$	1M	
$\approx \dfrac{0.840896414}{1.224744871}$		
$\theta \approx 46.6392948°$		
$> 45°$		
Thus , the claim is disagreed .	1A	f.t.
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学友社 Hok Yau Club 香港模擬文憑考試 17/18

數學科試卷二答案

(1) A	(11) D	(21) C	(31) A	(41) B
(2) B	(12) D	(22) D	(32) A	(42) A
(3) B	(13) C	(23) B	(33) B	(43) C
(4) D	(14) A	(24) C	(34) B	(44) C
(5) B	(15) A	(25) A	(35) C	(45) D
(6) C	(16) B	(26) D	(36) C	
(7) A	(17) D	(27) B	(37) D	
(8) D	(18) B	(28) A	(38) D	
(9) C	(19) A	(29) D	(39) C	
(10) C	(20) D	(30) A	(40) B	